Answer to the article « The fables of Ishango, or the irresistible temptation of mathematical fiction»

by Vladimir Pletser
Sciences Department, Manned Spaceflight and Operations Directorate,
European Space Research and Technology Centre, European Space Agency,
Noordwijk, The Netherlands

and Dirk Huylebrouck
Faculty of Architecture, University of Louvain, KUL Brussels-Gent, Belgium

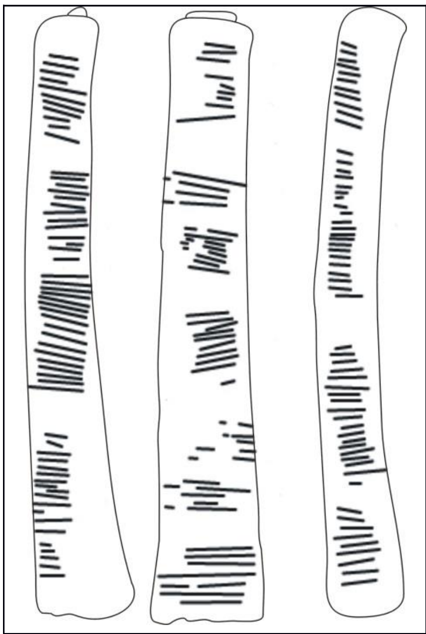
Our hypotheses

Since the end of the 1990s, the authors have introduced and explained the following hypotheses about the Ishango bone, discovered by archaeologist Jean de Heinzelin in the late 1950s:

1. The Ishango bone, dated from 22,000 years, can be considered as the oldest mathematical tool of humankind because the arrangement of the notches on three columns suggests an arithmetical intention.
2. In addition, it appears that several bases are used in this elementary arithmetic: the base 10 and base 12 with its submultiples 3, 4 and 6. The geometric arrangement of the notches in the various groupings on the three columns allows to compute other basic arithmetic operations.

These hypotheses were presented in several publications [1-18] and in International Congresses on mathematics and ethnomathematics.

The authors have explored in detail the arithmetic relationships between the numbers of notches. One knows for example that the sums of the numbers in the columns from left to right are 60, 48 and 60, that the numbers in the left column can be seen as primes, that doublings are seen in the middle column, and that the numbers in the right column can be interpreted as 10 and 20 plus or minus 1.



	3	11
11	6	
13	4	21
	8	
17	10 (9+1 ?)	
		19
	5 (1+4 ?)	
19		9
	5	
	7	

***Figure 1 :* Schematics of the first Ishango bone** *(based on a sketch Wikimedia Commons) and the corresponding numbers of notches of the three columns.*

Our conclusions

The authors have proposed various hypotheses, and rejected others (such as the one of prime numbers), yielding an unspectacular conclusion: it is probably a tool that counts events or things, noted by someone who mixed bases 10 and 12.

However, the authors did not enter into discussions about the use of this rod: is it an arithmetic game, the hypothesis of the archaeologist Jean de Heinzelin; or a calendar, according to Marshack; or a simple counting tool; or something else? The authors have left this discussion to ethnologists and anthropologists.

Following a *BibNum* analysis

Olivier Keller, in a *BibNum* Analysis [19], has criticized the work of the authors concerning this rod. It took them some time before they decide to respond to these criticisms. Indeed, the work of this critic has never been published in international journals or journals with peer reviewing. In addition, this critic has never participated in any international scientific congress, where he could have defended his point of view, which is part of the scientific process. Without wanting to polemicize, the authors want to present the following few arguments to refute these criticisms one by one.

Our résponses

1. The bone, the tool and its age

[…] *the first of the two Ishango bones – rose to fame by being presented as a noteworthy scientific text* […]

Strange formulation: a bone is not a «scientific text ».

A fragment of quartz affixed to one end shows that it was a tool handle […]

This hypothesis is supported by renowned archaeologists who studied the bone: Jean de Heinzelin (B), Alison Brooks (USA), John Yellen (USA), Els Cornelissen (B), and is also mentioned on the website of the Institute of Natural Sciences in Brussels where the object is exposed.

[…] *It is usually dated to 20,000 years BCE.*

The accepted dating is of 22,000 years. This dating is obtained by the carbon-14 method, and confirmed by several other archaeological methods, as explained by Alison Brooks. The Ishango civilization might even be 90 000 years old, following the work of J. Yellen [20].

2. The suspicion on the notches

Several of the notches are worn way or barely visible, which immediately makes any numeric interpretation suspect.

It would be appropriate to not rely on the observation of photographs or reproductions, but to consult the original work of J. de Heinzelin [21, pp 64-70] describing the circumstances of its discovery, unless of course one should also suspect the illustrious archaeologist of intellectual dishonesty. Moreover, the technician and collaborator of de Heinzelin, Marcel Spinglaer, was a recognized specialist who can certainly not be suspected.

Let's take the middle column: according to the author, 3 is doubled to 6, 4 to 8 and 5 to 10. But the 5 and the 10 are doubtful: one of the sets of 5 is genuinely illegible, and in reality the 10 could be a 9. In addition, in the case of a duplication of 5, 3 and 4, there is to explanation as to why the set of five notches is shown twice, whereas the group of three and five [sic] are shown only once.

Where is the problem? Couldn't we see just two operations of duplication (3 => 6 and 4 => 8) and an addition operation (10 = 5 + 5)?

And what is the role of 7, which is neither involved in duplication nor doubled? Unless the bottom of the middle column reads 10, 4, 5 and 7 (and not 10, 5, 5 and 7), which would give us 7 doubled, with 10+4, and 5 doubled, with 10.

Isn't the author of these lines engaging in speculation that he himself so strongly decries? Nevertheless, the proposed interpretation is interesting and we leave the responsibility to its author.

Once it has been decided that the sets of notches are numbers […]

It is true that the starting hypothesis of any mathematical interpretation of this Ishango rod is the association with a group of notches of the number of notches in this group. This critic himself supports this hypothesis in his publications [22, p. 569] and [23]: « Behind the enumeration using notches hides the number: the history of arithmetic begins. One will write it someday, thanks to the large number of monographs devoted to the numeration of the primitives, and one will highlight two main discoveries that we owe to our illiterate ancestors: the number and the systems of numbers, i.e. the number and the calculation. »

[…] *it's easy – given a few arrangements here and there – to load the bone with meaning, or even, if one pursues the argument a little further, as above, to make it say contradictory things.*

Of course, and Ramsey theory [24] tells even more about any series of numbers, but let's stick to the «few arrangements here and there». One could see here (as originally suggested by de Heinzelin) primes, or Pythagorean triplets, etc. However, if we care to consider all groupings at once and look, not by the small end of the spyglass[1], but at the whole, one can only be led to consider an arithmetic hypothesis. J. de Heinzelin was very careful about this [21, pp 67-70]: «... In the view of the mathematicians I consulted, no logical way can prove that these numbers are due or not to the kind of 'chance' that occurs for example in a hunting or revenue account. However, everyone would dare to say, I think, in the explanation of this table of numbers, that the feeling of human things is tipped for the arithmetic hypothesis. If there is arithmetic,

1. [translator's note] The French expression stating "*regarder par le petit bout de la lorgnette*" (translating literally as 'looking by the small end of the spyglass') means in fact not seeing the whole, but concentrating on a tiny part and missing the big picture.

calculations are certainly based on bases 2 and 10; the primitive use of these is not surprising, because they are the most natural to man." The nine renown scientists of the time whom de Heinzelin had consulted include L. Hogben, author of the book *Mathematics for the Million* [25], and specialists of the Free University of Brussels, and of other universities.

The interpretation of the numbers 11, 13, 17 and 19 of notches on the left column of the rod as a sign of a knowledge of prime numbers was rejected by the authors in several publications. However, the disposition of the numbers 11, 13, 17 and 19 called for another explanation, for example by considering them respectively as 12 ± 1 and 18 ± 1. One then get indications pointing to the base 12 (and/or its submultiples 3, 4 and 6). In addition, the sums of three columns are multiples of 12. Bases 10 and 20 are obvious considering human anatomy of hands and feet, but the base 12 is just as well. One counts with the thumb of one hand the phalanges of the four fingers, as it is still practiced today by some populations, and the number of dozens on the other hand, giving a total of 60. This interpretation suggests why 12 and 60 often go together, even today, as for the time subdivision.

3. A Slide Rule?

Indeed, they go further than their predecessor by affirming that the numbers in the three columns are related in such a way that together they form a calculation rule [sic].

It was never stated that the Ishango bone was a slide rule. The authors describe in their publications the arrangement and the geometrical characteristics of the notches in the various groupings and hypothesize that this rod could have served as support for a method of counting, similar to a slide rule, without being necessarily such an instrument. For example, [7, pp 342]: « [...] the proposed hypothesis of considering the bone as an ancient 'slide rule' to display simple addition arithmetic fits well with the various notch geometrical patterns. It shows a systematic occurrence of the derived base 12 and the central role that this base played in the proto-mathematics of the ancient Ishango people. »

4. Invented additions?

Only four of the additions are exact. However, as the authors want this bone to be an addition chart, they have to forcibly make up others. For example, the 3 and the 6 in the middle column, they tell us, are almost aligned with the 11 in the right column, ergo the 3 and the 6 have been *added together and the answer shown on the right. True, the answer is out by two (cf. the +2 in the table above), ergo the 2 has been left out for some unknown reason! Pletser and Huylebrouck use the same technique to invent three other additions, shown in the second and last two lines of the table above, with the missing numbers in parentheses.*

Let's suppose for a moment that this is an addition table. What is the point of such a muddled table, whose numbers have to be considered now in sets of two, now in sets of three, and whose answers are sometimes shown on the right and sometimes on the left?

[and further] At the conference in 2007, the authors provided an additional chart and assumed that the fifth number in the middle column is 10. This time, all the operations are incorrect.

The authors tried to consider all possibilities of combinations of operations of elementary arithmetic between the notch numbers of the middle column and those of the other two columns, while respecting the arrangement and shapes of the notches [15]. Some attempts gave a result, others did not. Two of these attempts were presented in these tables, with no malicious intent.

And what is the point of additions where, for example, amalgamating three sets of 3, 6 and 4 into a single set of 13 does nothing but make the number more difficult to understand?

Only that it brings a visual illustration of the sum of the three sets of 3, 6 and 4 notches on the middle column as a new set of 13 notches in the left column.

5. Intention of prehistoric men and unknown.

It is well known that such an "addition" would have been completely meaningless in the first true number systems. To return to the case in hand, the number 13 would never have been represented with 13 regularly spaced notches, but instead in distinct sets to make it easier to understand. In Ancient Egypt, for example, the hieroglyph 9 was not 9 equally spaced aligned bars, but either 4 bars placed below 5 other bars, or more often three sets of 3 bars placed atop one another.

Is it a criticism or an argument? The authors no longer understand this critic, because on one hand, he decries their similar interpretations, and on the other hand, he proposes himself the bold idea of the passage of a representation of a set of 13 notches to a more abstract representation of a number, 13. And that was certainly not the intention of prehistoric men of Ishango, who had probably not yet discovered the concept of numbers and their representation.

This passage to abstraction and representation of the numbers is well posterior to the prehistoric civilization of Ishango.

In addition, this critic points out in his footnote 9 of the page 9 of [19], the Babylonian numeration using a symbol (the nail) in base 10 and another (the chevron) in base 6. The number 9 is then written with nine nails, according to B. Rittaud [26, p.2], quoted by this critic. So what: two weights, two measures?

> *It doesn't work, so let's invoke unknown intentions! These are joined by some rather unconvincing speculations about the comparative length or gradient of the notches, which in any case do not justify the necessary addition of 2, 1 or -1 to lend a semblance of coherence to the whole.*

Regarding the « unknown intentions » in the first sentence, the authors signal that it happens also to this critic himself to confess his ignorance, for example [27, p. 73]: « For the reasons that the author ignores, the primitive symbols are essentially geometric symbols. » In addition, these « unconvincing speculations » refer to the study of the shape and size of the notches in the various columns that are detailed in [15].

6. The choice of numbers on the bone

> *As for the fifth number in the middle column, one might well ask why the authors choose 10, which does not give a single correct answer, rather than 9, which gives four correct answers. The reason is that with 10, the total of the middle column is 48, which is a multiple of 12, like the total 60 in the left and right columns.*

Indeed.

> *To account for the choice of numbers on the bone, Pletser and Huylebrouck posit that: « The numbers 3 and 4 could have formed the base of the arithmetic system used by the ancient Ishango people for operations on small numbers and that the derived base 12 was used for larger numbers. » Where are the bases 3 and 4? In the middle column, according to the authors, because from top to bottom it shows:*
>
> *- 3 then 6, so 3 then 3 x 2*
>
> *- 4 then 8, so 4 then 4 x 2*
>
> *- 9 or 10, so 4 x 2+1 or 4 x 2+2*
>
> *- two times 5, « showing two ways of obtaining the composed number 5, in adding 1 or 2 to either of the bases 3 and* [sic] *4 »*
>
> *- 7 « showing how to obtain the composed number 7 by adding the two bases 3 and 4 ».*

Indeed.

7. Combination of several bases

But if 5 is shown twice because there are two bases, why are the other numbers – 6, 8, 9 or 10 and 7 – shown only once? Furthermore, if the aim had been to demonstrate a base, this would have been clear to see. Two groups of 3 should be visible within a set of 6, two groups of 4 should be visible within a set of 8, and so on and so forth. Yet there is nothing of the kind. No regular groupings can be detected that are suggestive of a base.

Nowhere was it said that the sculptor of notches had "the aim to demonstrate a base" on this bone. The authors do not understand where this critic's remark comes from. The authors suggested that this rod is probably the testimony of a people who was counting by mixing bases 10 and 12 (or 6). A European would maybe write ||||| ||||| |||| to count 14 days, whereas a man from Ishango may write |||||| ||||||| ||. The combination of several bases is common and is not surprising. For example, in French, 77 and 93 refer to a past in bases 10 and 20. If in 22,000 years, one will find a French text with the expressions 'seventy' and 'eighty-three[2]', the archaeologist from the future might conclude with reason that the French language was mixing bases 10 and 20. In addition, there would be little chance that this text would be a scholarly dissertation on arithmetic explaining the use of these different bases.

Then, the duplication is not necessarily an operation of addition to the original of a copy of the original, or a simple multiplication by 2. It may be a reconstruction of a new group with a number of notches double of the initial group but arranged differently [15]. Requiring « to see clearly two sets of 3 within the set of 6, two sets of 4 within the set of 8 and so forth » is what might be called 'looking through the small end of the spyglass and demonstrates a too simplistic modern mathematical-cultural egocentricity.

Where is the base 12? On one hand, as we have already noted, in the column totals, which are multiples of 12. And on the other hand, according to the authors, in the fact that in the middle column:

- 6 is involved in two assumed additions (first two lines of the above table): 2 x 6 = 12

- 4 is involved in three assumed additions: 3 x 4 = 12

- 8 is involved in three assumed additions: 3 x 8 = 24 = 2 x 12.

2. [translator's note] In French, the number 83 is said and written literally 'four-twenty-three' ('quatre-vingt-trois').

Indeed, but, as mentioned above, the base 12 also appears in the left column with the numbers 11, 13, 17, 19, which, after the abandonment of the assumption of the prime numbers, are seen as 12 ± 1 and 18 ± 1.

This gives us the following situation: 6 is incised once as a group of six notches in the middle column. But, because we have assumed that it is involved in two additions, and even though these are incorrect (vide the unknown intention), that gives 12! The same goes for 4 and 8, which supposedly appear three times each, giving 12 and 2 x 12 respectively. How can one possibly be convinced by such sleights of hand as these, which are worthy of the most insipid numerological tract?

Let's put things in their place. The bases 3 and 4 are deduced from the basic operations in the middle column and the base 12 is deduced from the numbers 12 ± 1 and 18 ± 1 of the left column. Simply, the authors find that the numbers of the middle column are equal to 12 or multiples or submultiples of 12. This fact is nowhere used as an argument to reinforce the veracity of the hypothesis of the base 12 and its submultiples.

8. The second bone

In 1959, Jean de Heinzelin found another notched bone, again in Ishango. In 1998 he put forward an interpretation of these notches which, according to Pletser and Huylebrouck, confirms the above. However, there is little point in continuing to test the reader's patience with this matter.

The authors doubt that this critic has seen the second rod, because nothing had been published on this subject prior to 2007, on the occasion of the Congress to which this critic was invited, but that he preferred to avoid.

The authors believe instead that this point is of great interest. On his deathbed, Jean de Heinzelin had changed his mind about the interpretation of the notches of the first rod, actually seeing a basic arithmetic tool using several simple bases. This conclusion had not been published according to his last wishes, and could be published only some time after his death. The authors were very surprised to see their hypothesis confirmed a posteriori by de Heinzelin, at the same time as the announcement of the existence of the second bone.

A glance at Figure 5 and the following passage will be edifying enough:

« Prof. De Heinzelin added that the minor on the E Column is at the "10-spot", and wondered if this announced "a passage from the base 10 to base 12" […] Since the C column has a total of 20 carvings, and the E column 18 […] the bases 6 and 10–20 seem to emerge. Moreover, there are two spatial concordances between the rows, at E10 = F1 = G10 and at E12 = F2 = G12.12. » According to de Heinzelin's hypotheses, which were

later taken up by Pletser and Huylebrouck, this bone may have borne witness to a change of base, or had a didactic function, or even played a role in exchanges between different ethnic groups, some of which used base 10, while others used bases 12 or 16 among others.

The archaeologist de Heinzelin was a world leader in archaeology. His arguments were always based and had weight.

9. The small end of the spyglass

But remember, the base argument can be taken seriously only if the groupings are clear and systematic. 18 = 3 x 6 is not proof of base 6!

And remember also that this way of seeing things is contrary to the scientific approach which presupposes an open mind and the examination of all possible and plausible hypotheses. Looking only for « clear and systematic groupings » is so reducing and simplistic, and is evidence of a narrowness of view (the small end of the spyglass...) passing any observation through the mould of our modern and current vision of things without questioning the way in which a prehistoric man could have seen things from his point of view.

Indeed, 18 = 3 x 6 is not enough to deduce the existence of a base 6 if there is only one indication of this type. On the other hand, if several indications are present on the first rod of Ishango as shown by de Heinzelin and other scientists, with several examples of basic operations of duplication and addition involving the numbers 3, 4 and 12, next to the number 10, any scientist who has been trained in the scientific method, namely observation and deduction, is entitled to ask the question if there is actually more than a simple chance and if correlations between the different bases should not be considered.

10. Ethnographic comparatism

The handful of ethnographic examples adduced by the authors of the conference proceedings are just as unconvincing. The fact that the people of the Congo say the equivalent of "twelve-one" when they mean thirteen makes base 12 relevant here, but what it actually signifies is that the only way to say 13 is 12, then 1. The visual equivalent would be to incise a set of 12 marks followed by a space and a separate notch. The same goes for peoples who use their fingers to represent numbers. The Shambaa of Tanzania represent the number 6 by stretching out three fingers on each hand, and say the equivalent of "three- three" for six. Number 8 is "four-four" and represented by four fingers on each hand. Number 7 is more complex, in that it is pronounced as "ten minus three" and represented by four fingers on the right hand and three on the left hand. The gestures for the three numbers – 7, 8 and 6 – make a clear distinction between bases

3 and 4, if the term base is indeed appropriate here. Yet such a clear and systematic separation into subsets of 3, 4 and 12 notches is not in evidence on either of the Ishango bones. These ethnographic examples only make matters worse for Pletser and Huylebrouck's theories.

It is surprising to read the beginning of this excerpt from the pen of someone who presents himself as the promoter of the ethnographic comparatism, and who is even ecstatic [22, pp. 563-564]: « The ethnographic comparatism, which recognizes this analogy, opens a research path barely explored for the mathematics of prehistory, but rich of promises since it allows, to a certain extent, to make talk the archaeological findings. And it also imposes on the assumptions and theories based on the prehistoric documents to be verified by the ethnographic documents and vice versa. »

The authors do not see how or why « These ethnographic examples only make matters worse for the theories » of the authors. There is nothing that allows to say this, as this critic does. On the contrary, if he had bothered to read carefully the different ethnographic circumstantial arguments given in [15].

11. The mathematical illusion

This is what we have just seen with the speculations of Heinzelin, Pletser and Huylebrouck, who in fact are only the latest in a long line of victims of mathematical illusion. This illusion is all the more alluring and persistent when prehistory is involved.

An attempt of hurtful words that calls for no comment.

12. A question of battered women?

Yet, whether simple or sophisticated, such purported interpretations all have the same arbitrary foundation: the belief that notches are necessarily numerical. Claudia Zaslavsky recounts that some African women occasionally make a notch on the handle of their wooden spoon. Are they marking the passing days? Or playing with numbers? Not all [sic]*: they make a notch each time their husband hits them, and when the spoon handle is full, they ask for a divorce.*

The authors cited Zaslavsky in their reference 23, p. 166 [15]. More even, they cited similar reports written by missionaries and administrators of the Belgian Congo who described how and why such rods with notches were used.

In addition, it is true that this hypothesis may not be entirely rejected. But then, why an Ishango prehistoric woman would have made marks so sophisticated with groups that provide such regularity in the whole? And moreover, why make marks on three columns? One can imagine that she could have already asked for a divorce after filling the middle column with 48 notches, or the left or the right column, with 60 notches each time!

Although this hypothesis cannot be formally rejected, it is clear that this hypothesis would be the least likely of all for the first rod of Ishango.

> *A notch may be nothing more than a mark, which seems like small fry if one is obsessed with arithmetic. And yet that is the most important invention we owe to our ancestors of the Upper Palaeolithic* [sic]*: the sign.*
>
> Indeed.
>
> *By losing ourselves in haphazard mathematical speculations, we waste time, money and paper, and this when there is so much to discover about prehistoric signs – including the intellectual gestation of the concept of number – by considering them alongside the ethnographic records.*

If this critic had followed this recommendation to the letter, his scientific work, confined to a few publications at the national level and repeating the same criticisms as those discussed here, would have disappeared entirely.

The two ethnographic counterexamples given by this critic [19, pp.14 - 17] are excellent but are not counter-examples, on the contrary. The authors took similar examples in some of their publications. These stories show especially an important point: the a priori knowledge of the reason for being, or *raison d'être,* of these marked or signed media. In other words, another source of information (written, oral tradition, additional information, whatever the medium) allows an easy decryption of these objects.

The Ishango bones have been found with other tools and spear tips [21]. None of these other artefacts indicate any link with the notches on the bones. The critic says himself [22, p. 567] speaking of the Ishango bone:

> *Similarly, no ethnographic document allows to support the above-mentioned thesis [...] of a hunter-gatherer producing a table of primes, a table of doubles, ...*

So what to do? Ignore these artefacts and leave them in a drawer or on the contrary study them and try to decipher them? Remember that the study of the first bone was done without even knowing that a second bone had been discovered in the same place and that the description of this second bone has been published several years after the death of its discoverer. Yet the conclusions drawn on the first bone alone by the authors have been corroborated afterward by the conclusions of Jean de Heinzelin before the publication of the authors' results and without knowledge of de Heinzelin conclusions.

13. Which calculator?

The currency of such fictions as the "Ishango calculator" – and the fact that they are often taken at face value – is a sorry state of affairs, not only because of their intrinsic flimsiness and implausibility, but also because the archaeological and ethnographic archives could be put to so much better use.

The authors have never used the term "Ishango calculator" in their publications and communications. They leave the responsibility and paternity of this expression to this critic.

Conclusions

1. A appropriate conclusion

The study of the first bone led to an appropriate conclusion, although not spectacular. It was reached without knowing that a second bone had been discovered in the same place and before the publication of its description several years after the death of its discoverer. Yet the conclusions that the authors drew on the first bone have been corroborated *a posteriori* by Jean de Heinzelin's conclusions on his deathbed before the publication of the results of the authors and without knowledge of de Heinzelin's conclusions.

2. An epistemological fault

A better bibliographic research would have allowed this critic to forge his opinion on all the elements, but one can doubt that this critic has a sufficient command of the English language, as he pulls items out of their context and uses second hand sources instead of the original documents. Well, it is true that this is not the first time that this critic has problems to read scientific publications and to document himself correctly (see for example [28]).

3. Breach of copyright

The authors also signal the use by this critic of images without respect for copyright laws; his illustrations are taken from other articles without any authorization request, or from the site of the Museum of Natural Sciences in Brussels, without respect for applicable regulations.

4. A doubtful, little academic vocabulary

The authors can't resist the pleasure to highlight in this little pamphlet the adjectives and expressions which shed light on the style of this critic: "ridiculous" ([19], p. 5), "they have to forcibly make up others" (p. 6), "a muddled table" (p. 7), "It doesn't work" (p. 8), "unconvincing speculations" (p. 8), "a semblance of coherence" (p. 8), "which are worthy of the most insipid numerological tract?" (p. 9), "this "trick" doesn't work" (p. 9), "Since we *need* to have 12s" (p. 9), "such cobbled-together computations" (p. 9), "to test the reader's patience with this matter. A glance at Figure 5 and the following passage will be edifying enough" (p. 10), "Next comes the invention of some kind of concrete context and a story. In the end, what we have before us is mathematical fiction." (p. 11), "speculations" (p. 11), "the latest in a long line of victims of mathematical illusion" (p.11), "The siren song of mathematical illusion" (p. 11), "a naïve mathematician" (p. 11), "Four is feminine because women have four lips." (Note 24, p. 14), "calamitous enough for mathematical fablers" (p. 15).

As for the terms "intrinsic flimsiness and implausibility" (p. 16), they would be better attached to this little critical pamphlet.

The authors deplore this language but they do not want to get into a pointless controversy for reasons explained in the introduction.

However, they are open to debating ideas and they would be happy to participate in an exchange of views at a conference.

■

(March 2016)

(translated in English by the authors, published January 2017)

References of the article of MM. Pletser and Huylebrouck

– [1] D. Huylebrouck, 'Puzzles, patterns, drums: the dawn of mathematics in Rwanda and Burundi', *Humanistic Mathematics Network*, Journal no 14, November 1996.

– [2] D. Huylebrouck, 'The Bone that began the Space Odyssey', *The Mathematical Intelligencer*, Vol. 18, no 4, p. 56, Autumn 1996.

– [3] D. Huylebrouck, 'Histoires célestes du Rwanda' (Celestial stories in Rwanda), Ciel et Terre, *Bulletin of the Royal Belgian Soc. of Astronomy, Meteorology and Geophysics*, Vol. 113 (2), 83-89, 1997.

– [4] D. Huylebrouck, 'Counting on hands in Africa and the origin of the duodecimal system', *Wiskunde en Onderwijs* (*Mathematics and Education*), no 89, 1997.

– [5] V. Pletser, D. Huylebrouck, 'The Ishango Artefact: the Missing Base 12 Link.' Proc. Katachi Univ. Symmetry Congress (KUS2), T. Ogawa, S. Mitamura, D. Nagy & R. Takaki (ed.), Paper C11, Tsukuba Univ., Japan, 18 Nov. 1999.

– [6] V. Pletser, D. Huylebrouck, 'Research and promotion about the first mathematical artefact: the Ishango bone', Proc. PACOM 2000 Meeting Ethnomathematics and History of Mathematics in Africa, Cape Town, South Africa, 1999.

– [7] V. Pletser, D. Huylebrouck, 'The Ishango Artefact: the Missing Base 12 Link', *Forma*, 14-4, 339-346, 1999.

– [8] D. Huylebrouck, 'Afrika + Wiskunde' (in Dutch), VUBPress of the Free University of Brussels, 2005; translated in French 'Afrique + Mathématiques' VUBPress of the Free University of Brussels, 2008.

– [9] D. Huylebrouck, 'De parabolische vlucht van het Ishangoartefact, de oudste wiskundige vondst' (The parabolic flight of the Ishango artefact, the oldest mathematical discovery), *Wiskunde en Onderwijs (Mathematics and Education)*, n° 101, 2000.

– [10] D. Huylebrouck, F. Dumortier, 'A Stamp for World Mathematical Year', *European Mathematical Society Newsletter,* p. 27, November 2000.

– [11] D. Huylebrouck, 'Middle column of marks found on the oldest object with logical carvings, the 22000-year-old Ishango bone from the Congo', sequence A100000, On-Line Encyclopaedia of Integer Sequences, http://oeis.org/, November 2004

– [12] D. Huylebrouck, 'L'Afrique, berceau des mathématiques' (Africa, the craddle of mathematics), *Pour la Science*, Dossier 47, April-June 2005 (6 pages). The article was translated in Portuguese (in Brazil) and in German.

– [13] D. Huylebrouck, 'Mathematics in (central) Africa before colonisation', Anthropologica et Praehistorica, *Bulletin of the Royal Belgian Association for Anthropology and Prehistory*, 135 – 162, Vol. 117, 2006.

– [14] D. Huylebrouck, 'Report: The ISShango project', *Journal for Mathematics and the Arts*, Taylor and Francis, September 2008.

- [15] V. Pletser, D. Huylebrouck, 'An interpretation of the Ishango rods', Proc. Conf. 'Ishango, 22000 and 50 years later: the cradle of Mathematics?', D. Huylebrouck ed., Royal Flemish Academy of Belgium, KVAB, 139-170, 2008.

- [16] D. Huylebrouck, 'Foreword to the book Lusona: find the omitted drawings' by Paulus Gerdes (Mozambique), Lulu Editions, July 2009.

- [17] D. Huylebrouck, R. Matteus Berr, 'Vermessung', in 'Kunstlerhaus catalogue EVO EVO! 200 Jahre Charles Darwin', I. and P. Braunsteiner eds, Künstlerhaus ISBN 978-3-900926-84-7 Verlag Lehner Wien, 2009.

- [18] D. Huylebrouck, 'België + wiskunde' (Belgique + Mathématique), Academia Press Gent, Belgique, 2013.

- [19] O. Keller, « The fables of Ishango, or the irresistible temptation of mathematical fiction », *Bibnum*, August 2010.

- [20] J. Yellen, 'Ishango and its importance for later African prehistory', Proc. Conf. 'Ishango, 22000 and 50 years later: the cradle of Mathematics?', D. Huylebrouck ed., Royal Flemish Academy of Belgium, KVAB, 63-81, 2008.

- [21] J. de Heinzelin de Braucourt, 'Exploration du Parc National Albert', Fascicule 2, Les Fouilles d'Ishango, Instituts des Parc Nationaux du Congo Belge, Bruxelles, 1957.

- [22] O. Keller, « Questions Ethnographiques et Mathématiques de la Préhistoire », *Revue de synthèse* : 4e S. n° 4, oct.-dec. 1998, p. 545-573.

- [23] O. Keller, « Préhistoire de l'arithmétique, La découverte du nombre et du calcul», in 'Si le nombre m'était conté', I.R.E.M. - Histoire des mathématiques, Paris, Ellipse, 2000.

- [24] F.P. Ramsey, 'On a Problem of Formal Logic', *Proceedings London Mathematical Society*, s2-30 (1): 1930, 264–286. doi10.1112/plms/s2-30.1.264

- [25] L. Hogben, « Mathematics for the Million», London, Allen & Unwin, 1936. ISBN-13: 978-0393310719

- [26] B. Rittaud, « À un mathématicien inconnu ! », *Bibnum*, novembre 2008.

- [27] O. Keller, « Préhistoire de la Géométrie: Premiers éléments d'enquête, premières conclusions », *Science et Techniques en Perspective*, Vol. 33, Université de Nantes, 1995.

- [28] J. Høyrup, 'Book review of 'Préhistoire de la Géométrie: Premiers éléments d'enquête, premières conclusions' , *Isis* 87,1 996, 713-714.

Annex notice by BibNum regarding Conclusions 3, p. 13 above:

Illustrations in *BibNum* article [19] by O. Keller come from scientific publications – one might consider that they are free of rights to illustrate another scientific article related to the some subject (the origin of the illustrations being obviously indicated).