The fables of Ishango, or the irresistible temptation of mathematical fiction

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***Figure 1:* Map of modern-day Uganda.** *Ishango is located in the Democratic Republic of Congo (a former colony of the Kingdom of Belgium), close to the border with Uganda and situated on the northern shores of Lake Edward (its approximate position is shown by the red square).*

Prehistoric artefacts are silent by definition, imbued with a delectable air of mystery. They are captivating and, we feel, rich in meaning. We naturally try to "make them talk", but it is hard to know if they have something to say and, if so, to make them speak the truth. All manner of fictionalised anachronisms lie poised for ambush, and one can very easily get caught out, especially when they are camouflaged … as mathematical truths. Mathematics, it is true, naturally look like definitive and timeless definitive truths. To top it off, almost any marking can be described or interpreted in mathematical language. The researcher acting in good faith can get carried away despite himself, yielding to the siren song of anachronism with the firm conviction that he is speaking the truth, for "this is mathematics"! The two perpendicular and symmetrical planes of a biface (hand axe),[1] it will be argued, show that the maker of the tool had mastered the

1. A tool that was mass produced by *Homo erectus* and *Homo ergaster* during the Lower Palaeolithic, starting from 1.5 million years ago.

Euclidian conception of space. Similarly, a given group of *n* strokes or notches will invariably be taken as number *n*.

We offer the reader a typical case study of this phenomenon, and one that is rather unusual on *BibNum*. Instead of analysing a noteworthy scientific text, we will show how, through mathematical illusion, a prehistoric artefact – the first of the two Ishango bones – rose to fame by being *presented* as a noteworthy scientific text. Discovered in 1957 by the Belgian archaeologist Jean de Heinzelin, the object's fame is now assured. It is on display at the Royal Belgian Institute of Natural Sciences, in Brussels, and presented to visitors as "humanity's oldest calculator". To mark the International Year of Mathematics in 2000, the Belgian Post Office published a stamp alluding, among other things, to this famous bone. The stamp shows a series of three then six vertical strokes at its base.

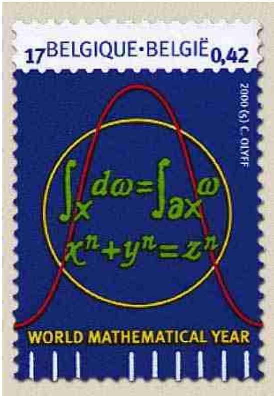

***Figure 2:* Stamp issued in 2000 by the Belgian Post Office to mark the International Year of Mathematics.** *The vertical strokes are an allusion to the notches on the Ishango bone.*

In 2007, an international conference entitled "Ishango, 22000 and 50 Years Later: The Cradle of Mathematics?"[2] was held in Brussels, and on 28 February, under the title "The incised bones of Ishango and the birth of numeration in Africa" [Les os incisés d'Ishango font naître la numération en Afrique], *Le Monde* reported that:

> *[…] they could represent the oldest evidence of humanity's mathematical capacities, fifteen millennia before numeration emerged at the same time as writing in Mesopotamia (modern-day Iraq).*

2. "Ishango, 22000 and 50 Years Later: The Cradle of Mathematics?"

Figure 3: ***The first Ishango bone seen from two sides.*** *On the left: the so-called "middle column" containing, from top to bottom and depending on whose interpretation it is, groups of 3, 6, 4, 8, 9 (or 10), 5 and 7 notches. On the right: part of the so-called "right column" (11 notches above and 9 below) and the "left column" containing, from top to bottom, groups of 11, 13, 17 and 19 notches.*
(Photographs by the Royal Belgian Institute of Natural Sciences)

The Ishango bone, it seems, is now a permanent fixture in both mathematical history books and popular science magazines, which venerate it as the oldest, or at least one of the oldest, records of human scientific endeavour. It can even inspire truly cosmic lyricism, for, as speakers at the 2007 conference explain:

> *[…] in Belgium, media coverage began in 1996 when Dirk Huylebrouck wrote* The Bone that Began the Space Odyssey *in* The Mathematical Tourist*, and continued with attempts to send the bone into space in homage to the Central African contribution to the development of technology.*[3]

3. Els Cornelissen, Ivan Jadin and Patrick Semal. "Ishango, a history of discoveries in the Democratic Republic of Congo (DRC) and in Belgium", in Dirk Huylebrouck (ed.), *Ishango, 22000 and 50 Years Later: The Cradle of Mathematics?*, 28/02–02/03 2007, pp. 23–39. Brussels, Koninklijke Vlaamse Academie Van Belgie Voor Wetenschappen En Kunsten.

It is acknowledged that such a promising artefact, and one that arouses so much enthusiasm, deserves its place in an institution devoted to the history of science. Let's therefore consider what it is exactly, and examine from a technical, historical and methodological point of view the mathematical interpretations[4] that have made it famous.

THE ISHANGO BONES AND THEIR NUMERICAL INTERPRETATIONS

The first Ishango bone (Fig. 3) was discovered in the area after which it is named, in the Democratic Republic of Congo. A fragment of quartz affixed to one end shows that it was a tool handle. It is usually dated to 20,000 years BCE. Placed lengthwise (10 cm), it shows three rows of more or less parallel but asymmetrically grouped notches (Fig. 4). Several of the notches are worn way or barely visible, which immediately makes any numeric interpretation suspect.

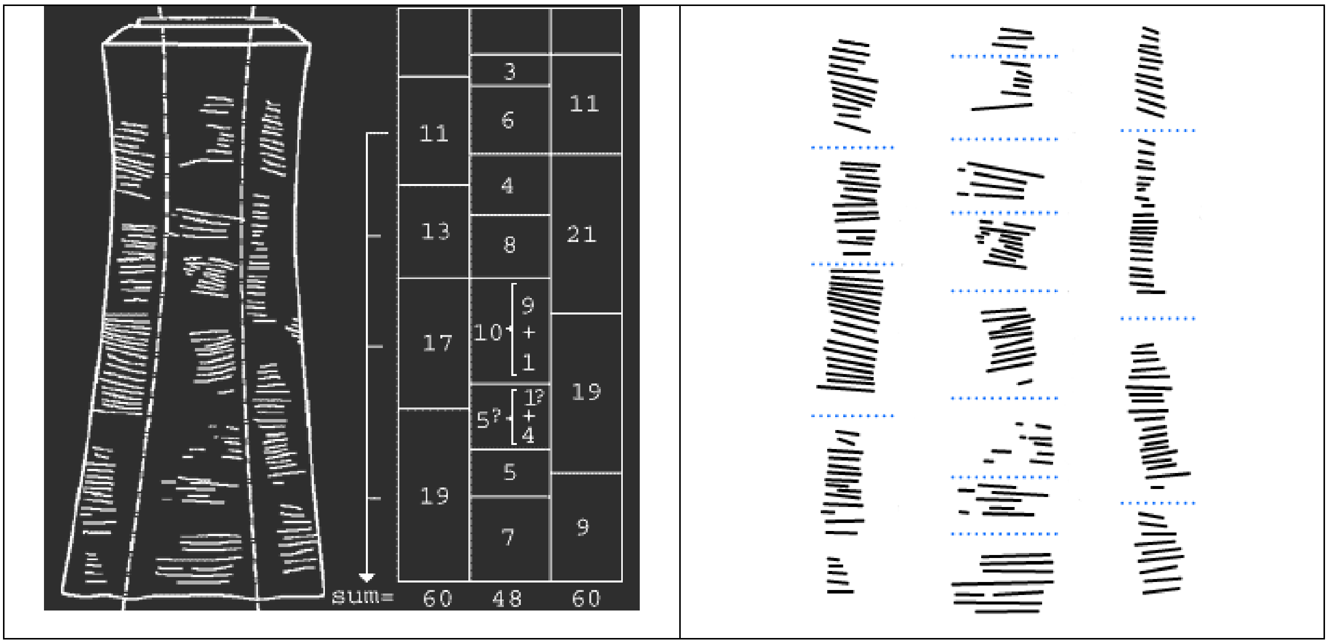

Figure 4: Full view of the first Ishango bone, with its three columns ("left", "middle" and "right"). *Source: Dirk Huylebrouck, "L'Afrique, berceau des mathématiques", in* Mathématiques exotiques, Pour la science *dossier, April/June 2005.*

Yet its inventor, Jean de Heinzelin, nevertheless took this risk and published his results in *Scientific American* in June 1962:

> *Take the first column, for example: 11, 13, 17 and 19 are all prime numbers (divisible only by themselves and by one) in ascending order,*

4. We will not consider Alexander Marshack's interpretation of the bone as a lunar calendar, which is now largely discredited.

and they are the only prime numbers between 10 and 20. Or consider the third: 11, 21, 19 and 9 represent the digits 10 plus one, 20 plus one, 20 minus one and 10 minus one. The middle column shows a less cohesive set of relations. Nevertheless, it too follows a pattern of a sort. The groups of three and six notches are fairly close together. Then there is a space, after which the four and eight appear – also close together. Then, again after a space, comes the 10, after which are the two fives, quite close. This arrangement strongly suggests appreciation of the concept of duplication, or multiplying by two.

It is of course possible that all the patterns are fortuitous. But it seems probable that they were deliberately planned. If so, they may represent an arithmetical game of some sort, devised by a people who had a number system based on 10 as well as a knowledge of duplication and of prime numbers.[5]

Let's take the middle column: according to the author, 3 is doubled to 6, 4 to 8 and 5 to 10. But the 5 and the 10 are doubtful: one of the sets of 5 is genuinely illegible, and in reality the 10 could be a 9. In addition, in the case of a duplication of 5, 3 and 4, there is to explanation as to why the set of five notches is shown twice, whereas the group of three and five are shown only once. And what is the role of 7, which is neither involved in duplication nor doubled? Unless the bottom of the middle column reads 10, 4, 5 and 7 (and not 10, 5, 5 and 7), which would give us 7 doubled, with 10+4, and 5 doubled, with 10.

In the left column, Heinzelin sees a list of prime numbers. If this is the case, the people of Ishango must have had a detailed understanding not just of simple duplication, but of the far more complex matter of multiplication in general, which would have made the two-times table on the side rather ridiculous, a bit like placing an addition table alongside a table of antiderivatives in a contemporary publication. And if the right column represents 10+1, 20+1, 20-1 and 10-1, why isn't the left column, for example, 15-4, 15-2, 15+2, 15+4 (instead of a list of prime numbers), in other words a kind of "arithmetical game" based on the average of the 10s and 20s in the right column? And here's another potential "game": on the left, the two end numbers total 30, as do the two middle numbers, whereas on the right the first and the third numbers total 30, and likewise the second and the fourth.

Once it has been decided that the sets of notches are numbers, it's easy – given a few arrangements here and there – to load the bone with meaning, or

5. Jean de Heinzelin, "Ishango", *Scientific American* 206, 105–116 (1962), doi: 10.1038/scientificamerican0662-105.

even, if one pursues the argument a little further, as above, to make it say contradictory things.

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The Ishango bone owes its fame not to the calculations of Jean de Heinzelin, but to the dedication and new calculations of two Belgian scientists, the mathematician Dirk Huylebrouck, and Vladimir Pletser of the European Space Agency. Though they reject Heinzelin's specific conclusions, they accept his premises, namely a purely mathematical interpretation. Indeed, they go further than their predecessor by affirming that the numbers in the three columns are related in such a way that together they form a calculation rule.

In an article published in 1999,[6] the authors put forward the first of several charts and suggest that in the middle column, the fifth number from the top equals 9 not 10. In the middle column, they take the groups of notches (again interpreted as numbers) first as sets of two, then as sets of three. They then add up and record the sum in the left column and in the right column. Here are the results:

Left column	**Additions in the middle column**	**Right column**
	3+6 (+2)	11
11	6+4 (+1)	
13	3+6+4	
	4+8+9	21
17	8+9	
	9+5+5	19
19	7+5+5 (+2)	
	7 (+2)	9

Only four of the additions are exact. However, as the authors *want* this bone to be an addition chart, they have to forcibly make up others. For example, the 3 and the 6 in the middle column, they tell us, are almost aligned with the 11 in the right column, ergo the 3 and the 6 have been added together and the answer shown on the right. True, the answer is out by two (cf. the +2 in the table above), ergo the 2 has been left out for some unknown reason! Pletser and

6. Vladimir Plester and Dirk Huylebrouck, "The Ishango Artefact: The Missing Base12 Link", *Forma*, 14, 1999, pp. 339–346.

Huylebrouck use the same technique to invent three other additions, shown in the second and last two lines of the table above, with the missing numbers in parentheses.

Let's suppose for a moment that this is an addition table. What is the point of such a muddled table, whose numbers have to be considered now in sets of two, now in sets of three, and whose answers are sometimes shown on the right and sometimes on the left? And what is the point of additions where, for example, amalgamating three sets of 3, 6 and 4 into a single set of 13 does nothing but make the number more difficult to understand? It is well known that such an "addition" would have been completely meaningless in the first true number systems. To return to the case in hand, the number 13 would never have been represented with 13 regularly spaced notches, but instead in distinct sets to make it easier to understand. In Ancient Egypt, for example, the hieroglyph 9 was not 9 equally spaced aligned bars, but either 4 bars placed below 5 other bars, or more often three sets of 3 bars placed atop one another.

At the conference in 2007, the authors provided an additional chart[7] and assumed that the fifth number in the middle column is 10. This time, all the operations are incorrect:

Left column	Additions in the middle column	Right column
	3+6 (+2)	11
11	6+4 (+1)	
13	4+8 (+1)	
	4+8+10 (-1)	21
17	8+10 (-1)	
	10+5+5 (-1)	19
19	5+5+7 (+2)	
	7 (+2)	9

The rationale remains the same. For the calculation in the first line, for example, the authors explain:

> *Why the additional adding of 2? No reason can be proposed but it seems that the relative positions of the notches of these three groups are not a coincidence and reflect an unknown intention.*[8]

7. Vladimir Plester and Dirk Huylebrouck, "An Interpretation of the Ishango Rods", in the aforementioned conference proceedings, pp. 139–170.

It doesn't work, so let's invoke unknown intentions! These are joined by some rather unconvincing speculations about the comparative length or gradient of the notches, which in any case do not justify the necessary addition of 2, 1 or -1 to lend a semblance of coherence to the whole.

As for the fifth number in the middle column, one might well ask why the authors choose 10, which does not give a single correct answer, rather than 9, which gives four correct answers. The reason is that with 10, the total of the middle column is 48, which is a multiple of 12, like the total 60 in the left and right columns.

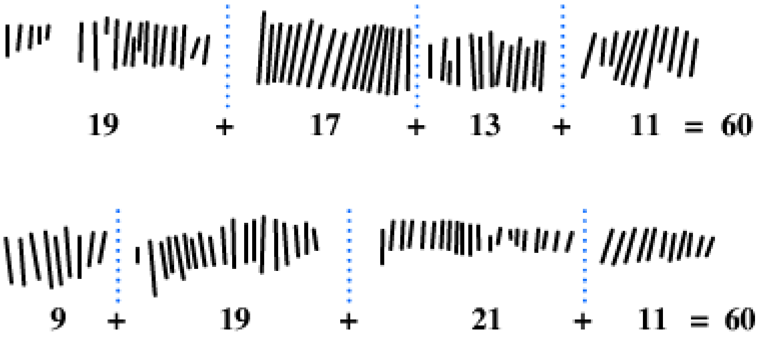

***Figure 5:* Supposed sum totals of the left and right columns (above and below respectively).**

To account for the choice of numbers on the bone, Pletser and Huylebrouck posit that:

> *The numbers 3 and 4 could have formed the base of the arithmetic system used by the ancient Ishango people for operations on small numbers and that the derived base 12 was used for larger numbers.*[9]

Where are the bases 3 and 4? In the middle column, according to the authors, because from top to bottom it shows:

- 3 then 6, so 3 then 3×2
- 4 then 8, so 4 then 4×2
- 9 or 10, so $4 \times 2+1$ or $4 \times 2+2$
- two times 5, showing "two ways of obtaining the 'composed' number 5, based on addition of 1 or 2 to either of the bases 3 and 4"[10]

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8. See Vladimir Plester, "Does the Ishango Bone Indicate Knowledge of the Base 12?", online: http://arxiv.org/ftp/arxiv/papers/1204/1204.1019.pdf.
9. *Ibid*. Note that Babylonian number system was sexagesimal (base 60) and used one symbol (wedge) for base 10 and another symbol (chevron) for base 6. See B. Rittaud's analysis of the YBC 7289 tablet on BibNum.
10. *Ibid.*

- 7, showing "how to obtain the 'composed' number 7 by adding the two bases 3 and 4"[11]

But if 5 is shown twice because there are two bases, why are the other numbers – 6, 8, 9 or 10 and 7 – shown only once? Furthermore, if the aim had been to demonstrate a base, this would have been clear to see. Two groups of 3 should be visible within a set of 6, two groups of 4 should be visible within a set of 8, and so on and so forth. Yet there is nothing of the kind. No regular groupings can be detected that are suggestive of a base.

Where is the base 12? On one hand, as we have already noted, in the column totals, which are multiples of 12. And on the other hand, according to the authors, in the fact that in the middle column:

- 6 is involved in two assumed additions (first two lines of the above table): $2 \times 6 = 12$
- 4 is involved in three assumed additions: $3 \times 4 = 12$
- 8 is involved in three assumed additions: $3 \times 8 = 24 = 2 \times 12$

This gives us the following situation: 6 is incised once as a group of six notches in the middle column. But, because we have *assumed* that it is involved in two additions, and even though these are incorrect (vide the *unknown intention*), that gives 12! The same goes for 4 and 8, which supposedly appear three times each, giving 12 and 2×12 respectively. How can one possibly be convinced by such sleights of hand as these, which are worthy of the most insipid numerological tract?

What's more, it turns out that this "trick" doesn't work with 10, 5 and 7. For example, 7, which is involved in two additions, would give 14 if one followed the method set out above. Never mind! Since we *need* to have 12s, we'll just take a 5 and a 7 from the two penultimate lines in the above table, then a 5 from the second-to-last line and a 7 from the last one. Even if we accept the authors' calculations, what use could they have been to our ancestors in Ishango? What is the point of such cobbled-together computations?

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In 1959, Jean de Heinzelin found another notched bone, again in Ishango. In 1998 he put forward an interpretation of these notches which, according to Pletser and Huylebrouck, confirms the above. However, there is little point in

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11. *Ibid.*

continuing to test the reader's patience with this matter. A glance at Figure 5 and the following passage will be edifying enough:

> *Prof. De Heinzelin added that the minor on the E Column is at the "10-spot", and wondered if this announced "a passage from the base 10 to base 12" […] Since the C column has a total of 20 carvings, and the E column 18 […] the bases 6 and 10–20 seem to emerge. Moreover, there are two spatial concordances between the rows, at E10 = F1 = G10 and at E12 = F2 = G12.*[12]

According to de Heinzelin's hypotheses, which were later taken up by Pletser and Huylebrouck, this bone may have borne witness to a change of base, or had a didactic function, or even played a role in exchanges between different ethnic groups, some of which used base 10, while others used bases 12 or 16 among others. But remember, the base argument can be taken seriously *only* if the groupings are *clear and systematic*. 18 = 3 × 6 is not proof of base 6!

[n.c.]

***Figure 6:* The second Ishango bone sketched by de Heinzelin.** *Source: Proceedings of the conference "Ishango, 22000 and 50 Years Later: The Cradle of Mathematics", ed. Huylebrouck, p. 166.*

The handful of ethnographic examples adduced by the authors of the conference proceedings[13] are just as unconvincing. The fact that the people of the Congo say the equivalent of "twelve-one" when they mean thirteen makes base 12 relevant here, but what it actually signifies is that the only way to *say* 13 is 12, then 1. The visual equivalent would be to incise a set of 12 marks followed by a space and a separate notch. The same goes for peoples who use their fingers to represent numbers. The Shambaa of Tanzania represent the number 6 by stretching out three fingers on each hand, and say the equivalent of "three-three" for six. Number 8 is "four-four" and represented by four fingers on each hand. Number 7 is more complex, in that it is pronounced as "ten minus three" and represented by four fingers on the right hand and three on the left hand. The gestures for the three numbers – 7, 8 and 6 – make a clear distinction between bases 3 and 4, if the term base is indeed appropriate here. Yet such a clear and systematic separation into subsets of 3, 4 and 12 notches is not in evidence on

12. Vladimir Plester and Dirk Huylebrouck, "An Interpretation of the Ishango Rods", *op. cit*.
13. Vladimir Plester, "Does the Ishango Bone Indicate Knowledge of the Base 12?", *op. cit.*

either of the Ishango bones. These ethnographic examples only make matters worse for Pletser and Huylebrouck's theories.

THE SNARE OF MATHEMATICAL FICTION

You can prove anything with statistics. It's an old adage, and one that should be extended to mathematics. Mathematics involves abstractions, and the problem with this is that they can be pinned on to any number of things.[14] Then, once they have been applied to a given context, we get carried away by the intrinsic rigour of the abstraction and *mistake a mere conceptual framework for actual substance*. If need be – and as we will see – the real-world artefact can be forcibly thrust inside the conceptual framework. Next comes the invention of some kind of concrete context and a story. In the end, what we have before us is *mathematical fiction*. This is what we have just seen with the speculations of Heinzelin, Pletser and Huylebrouck, who in fact are only the latest in a long line of victims of mathematical illusion. This illusion is all the more alluring and persistent when prehistory is involved. Take, for instance, the two Scottish engineers Alexander Thom (1894–1985) and his son Archie, who, using surveys of megalithic stone rows in Brittany and the British Isles, construed the latter as geometric constructions based on Pythagorean triples;[15] the influential mathematician B. L. van der Waerden (1903–1996) was taken in.[16] Or the Russian historian Boris Frolov, whose work on prehistoric graffiti on artefacts in Eastern Europe led him to conclude that there had existed various counting systems based on 3, 5, 7 and their multiples.[17]

Amateurs have even more on their plate. The siren song of mathematical illusion is never far away when it comes to prehistoric artefacts. In French museums in particular, dozens of incised bone or ivory sticks, dated to around 35,000–10,000 years ago, are just waiting for a naïve mathematician to elevate

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14. Jean-Pierre Adam play's on the various dimensions of a lottery ticket booth on avenue Wagram in Paris computed the distance between the sun and the earth, pi and Meton's cycle, among other things. Jean-Pierre Adam, *Le passé recomposé. Chroniques d'archéologie fantasque*. Éditions du Seuil, 1988.
15. For a detailed critique, see Olivier Keller, *Aux origines de la géométrie, le Paléolithique et le monde des chasseurs-cueilleurs*, Vuibert, 2004, pp16–18.
16. B. L. Van der Waerden, *Geometry and Algebra in Ancient Civilizations*, Springer, 1983.
17. B. A Frolov, "Comment on Alexander Marshack's paper", *Current Anthropology* (1979) 20(3): 605–607. By the same author: "Aspects mathématiques dans l'art préhistorique", paper delivered at the International Symposium on Prehistoric Art, Valcamonica, 1968. And "Les bases cognitives de l'art préhistorique", paper delivered at the Valcamonica Symposium, 1979.

them to the rank of scientific artefact and release them from the obscurity of their drawer.[18]

In 1937, Karel Absolon presented an 18 cm-long radial wolf bone in *The Illustrated London News*.[19] It had been discovered at Věstonice in the Czech Republic and dated to 30,000 years ago (Fig. 6). Fifty-five notches are incised on the bone, of which 25, the author tells us, are grouped into sets of 5. The object is therefore direct proof that prehistoric man could calculate. Yet no matter how hard one scrutinizes the photographs, there is not a group of 5 to be seen.[20]

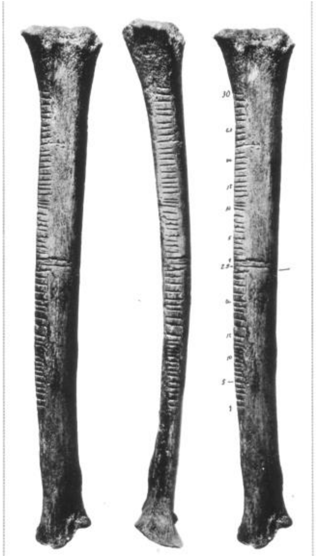

Figure 7:* Three views of the radial wolf bone published by Karel Absolon in The Illustrated London News *of 2 October 1937.

Another artefact made its entry in 1987. This time it was a baboon fibula dated to 35,000 years ago and bearing 29 notches, "which could lay claim to the title of the oldest known mathematical artefact".[21] The argument put forward was that its notches were similar to calendar markings used today by the Bushmen of Namibia. Of course, this very simple reading does not bear comparison with the sophisticated interpretation advanced for the Ishango bone. Yet, whether simple or sophisticated, such purported interpretations all have the same arbitrary

18. A large number of such sticks are reproduced in Marthe Chollot-Varagnac, *Les origines du graphisme symbolique. Essai d'analyse des écritures primitives en préhistoire*, Fondation Singer-Polignac, 1980.
19. "The World's Earliest Portait – 30,000 Years Old", 2 Oct. 1937.
20. Connoisseurs may be interested to know that this article also presents a fine bone needle with three distinct columns of regularly spaced notches, that is to say, all that is needed for it to be construed as an arithmetic instrument.
21. Naidoo Bogoshi, and Webb, "The Oldest Mathematical Artefact", *The Mathematical Gazette*, vol. 71, no. 458, Dec. 1987.

foundation: the belief that notches are necessarily numerical. Claudia Zaslavsky[22] recounts that some African women occasionally make a notch on the handle of their wooden spoon. Are they marking the passing days? Or playing with numbers? Not all: they make a notch each time their husband hits them, and when the spoon handle is full, they ask for a divorce. A notch may be nothing more than a mark, which seems like small fry if one is obsessed with arithmetic. And yet that is the most important invention we owe to our ancestors of the Upper Palaeolithic: the sign. By losing ourselves in haphazard mathematical speculations, we waste time, money and paper, and this when there is so much to discover about prehistoric signs – including the intellectual gestation of the concept of number – by considering them alongside the ethnographic records.

TWO ETHNOGRAPHIC COUNTER-EXAMPLES

The homes of the Bambara, Germaine Dieterlen tells us,[23] are decorated with "millet pulp" drawings on the walls of one of the rooms. One of these (Fig. 7) has rich fictional potential:

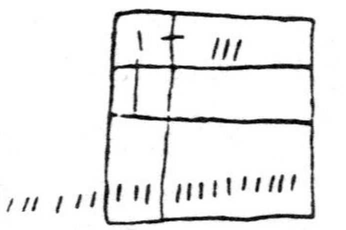

Figure 8: ***Ritual rectangle on a wall in a Bambara house*** *(Banankoroni, Mali). Source: G. Dieterlen (1988), p. 155.*

The numbers on the bottom row are 10 and 3. Ten divided by 3 gives 3 with a remainder of 1, and indeed it is 3 and 1 that are drawn on the top row. The vertical stroke in the middle row shows the link between the data in the bottom row and the result in the top row, and therefore confirm the aforementioned hypothesis. As for the 6, outside the rectangle, it is involved in two operations: 6+3 (top row)+1 (top row) = 10 (bottom row), and 6 = 3 (bottom row)+3 (top

22. Claudia Zaslavsky, *Africa Counts: Number and Pattern in African Culture*, Prindle, Weber and Schmidt, 1973.
23. Germaine Dieterlen, *Essai sur la religion bambara*. Brussels, Éditions de l'Université de Bruxelles, 1988, pp. 154–155. New edition (originally published by the Presses Universitaires de France in 1951).

row). In addition, the clear groupings into 3 confirm the existence of base 3 and the derived base 6.

Such is the fantasy that some would defend tooth and nail if the meaning of the figure were not known – if it had been discovered on the wall of a Palaeolithic painted cave, for instance. The reality is as follows: the three rows in the rectangle represent the three divisions of the universe – the sky and water, the air and the earth. Then,

> *In the upper left-hand square, a stroke connotes Faro in all his omnipotence as the sole master of the sky, water and life. A horizontal stroke recalls his domination over the world. Those that follow are his children. The total evokes the feminine form of genius.*[24] *[…] The central rectangle is the domain of Teliko: air and wind. The small vertical stroke through the left-hand square represents genius attempting to pierce Faro's plan and vanquish it. […] The lower rectangle delimits the earth, the seat of the Soba genii. The row of 22 sticks*[25] *covering it recalls the 22 elements of creation, the 22 things indispensable to man in this world.*[26]

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Let's now put our discernment to the test and consider the message sticks used by the Aborigines of Australia, which were described at the beginning of the last century.[27] These sticks or small planks bear regularly spaced and clearly grouped notches on their sides, making them ideal prey for the mathematical trap. The figure below (Fig. 8) shows one such message stick, with five and then ten notches on the right side, and eight, then four and then three notches on the left.

24. Four is feminine because women have four lips.
25. The figure shows only 19.
26. Dieterlen, *op. cit*.
27. Alfred William Howitt, *The Native Tribes of South East Australia*. London, Macmillan, 1904. Chapter XI.

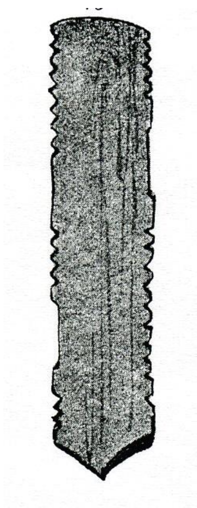

***Figure 9:* Message stick used by Australian Aborigines, with notches on each side.** *Source: Howitt (1904), p. 704.*

Lo and behold another fable, and one just as implausible as those we have considered in this article. As the total of each side is 15, what we have here, obviously, is a comparison of two bases. On the right, base 5 with 5 and twice 5; on the left, base 3 with 3 then 4, that is to say once 3 plus *one*, followed by 8, i.e. *twice* three plus *two*. What's more, as 8 (on the left) is almost opposite 5 (on the right), and 4 and 3 (on the left) are almost opposite 10 (on the right), "this cannot be the result of accident", and indeed there is a difference of 3 between both 8 and 5, and 4+3 and 10.

And here's the reality. This is an aide-memoire used by the messenger bearing a message for a particular group. The message is as follows:

> *I am here, five camps distant from you. In such and such a time I will go and see you. There are so and so with me here. Send me some flour, tea, sugar, and tobacco. How are Bulkoin and his wife and Bunda?*[28]

The aide-memoire works as follows:

> *Five notches represent the five camps (stages), distance to the recipient; a flat place cut on the stick shows a break in the message; ten notches the time after which the sender will visit his friend; eight notches the eight people camped with the sender; four notches the articles asked for; another flat place on the stick shows another break in the message, and three notches the three persons asked after.*[29]

There are many other examples of this kind. As these two are calamitous enough for mathematical fablers, we can rest our case.

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28. *Ibid.*, p. 695.
29. I*bid*.

AVENUES FOR RESEARCH

The currency of such fictions as the "Ishango calculator" – and the fact that they are often taken at face value – is a sorry state of affairs, not only because of their intrinsic flimsiness and implausibility, but also because the archaeological and ethnographic archives could be put to so much better use. Our examples, like all comparable examples from traditional peoples, show that markings can represent anything from individuals, to non-specified groups of individuals, to objects (tobacco, sugar, etc.), to distance to be covered on foot, or even an indeterminate plural. This is hardly enough to make them numbers. I don't need the number four if, when asked to enquire after Peter, Paul, Jack and John, I make four notches on a piece of wood to ensure I don't forget. This is a bijection, but a bijection is not a number.

And that, indeed, is the essential point: the common denominator of *all* the ethnographic artefacts of this kind is that they all show bijections, or an item-by-item symmetry between objects and signs. This presupposes the invention of the sign, i.e. a purely abstract representation that, visually speaking, is quite unrelated to that which is represented. Then – and this is a far subtler operation – the bijection itself must be invented, that is to say a one-to-one correspondence between entities and identical signs, the result of which is the reduction of given collection to an abstract collection of undifferentiated entities, i.e. a *plurality*. The individuality of these source entities (individuals, objects, days, etc.) is erased with these notches, strokes and dots: each becomes both identical to and distinct from the others. Identical yet different: this is the contradiction at the root of the concept of plurality. It implies that signs drawn should be as indistinguishable as possible, not only in appearance but also in how they are arranged. For example, three regularly spaced dots arranged on a line express this better than three dots arranged in a triangle. The notion of plurality implies erasing the specificity of objects, but an entity that has had every particularity erased is a *non-entity*, hence the tracing of signs that are as discreet as possible, or purely abstract marks: a non-existent "existant". In this sense, dots, lines and notches are obviously more expressive than mammoths, horses or complex geometrical signs.

The ethnography of hunter-gatherer societies makes it perfectly reasonable to hypothesise that hunter-gatherers of the Upper Palaeolithic invented plurality, and were therefore on the way to discovering number. This hypothesis does not require haphazard interpretations or calculations. The transition from plurality to number requires a sign system that fulfils three conditions:

- that among all the possible markings (dots, notches, body parts, etc.), some are designated as "standard collections"[30] with the specific function of expressing pluralities;
- that the comparison of two sets of objects is no longer direct – i.e. involving a one-to-one correspondence between the two sets – but mediated by the standard collection;
- that the standard collection is organised into an ordered sequence of degrees of quantity, for example:

  I, II, III, etc. for notches

  index finger, ring finger, middle finger, etc. for body parts

so that "III" and "middle finger", for example, convey both the cardinal (three) and ordinal (third) aspects.

Faced with the raw artefacts of prehistory, it is clearly impossible to know whether any of these conditions were or were not met. There is, however, hope of progress in this area, thanks to the work of Francesco D'Errico[31] among others. Using technical criteria rather than interpretative hypotheses, D'Errico and his team are attempting to determine whether the markings are decorative or not, and if they are not, whether we are dealing with an "artificial memory system", and if so, which one. But, at present, the most effective method involves the comparison of archaeological artefacts and ethnographic records.[32] As we have seen, such a juxtaposition is at the very least an effective safeguard against the temptations of mathematical fiction!

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30. I am borrowing this expression from the French mathematician Henri Lebesgue (1875–1941).
31. Francesco d'Errico, "A New Model and Its Implications for the Origin of Writing: The La Marche Antler Revisited", *Cambridge Archeological Journal*, vol. 5(2), 1995, pp. 163–206.
32. This is part of the author's current research for his forthcoming publication, *Préhistoire de l'arithmétique*.