The Pascaline,
the "machine that relieves the failure of memory"

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Pascal's precocity is well known. From the age of twelve he discovered the elements of Euclid, and at sixteen, he wrote his *Traité des coniques*.

Shortly afterwards, however, he would turn his attention to a far more practical concern. His father had just been appointed tax and tallage intendant for Basse-Normandie. Tax collection required many lengthy calculations, which, furthermore, had to be performed without the slightest error.

Pascal came up with the idea for an "arithmetical machine" capable of performing calculations without difficulty or risk of error. Over several years, he honed the machine to his satisfaction. In 1645, when he was twenty-three, he presented the machine to his contemporaries in an "*Avis nécessaire à ceux qui auront curiosité de voir la machine d'arithmétique, et de s'en servir"* ("Notice to those curious to see and use the arithmetical machine").

If the anachronism may be excused, this text is first and foremost an advertisement. Pascal has no qualms about extolling the merits of his machine. He asserts that the machine is robust and compact. It is simple and convenient to use, and the calculations are rapid, for despite the machine's robustness, its wheels are hand-operated ("the comfort of ease of movement"). What's more, the risks of error are nil. And, in fact, the advertisement is not at all misleading: these are very much real qualities, as demonstrated by the few examples of the machine that have been conserved.

The "Pascalines" today

Today only nine Pascalines – of an original manufacturing run of no more than twenty or so machines – are extant:

- Four are conserved at the Musée des Arts et Métiers (Paris), including the model donated by Chancellor Séguier, the French finance minister, and one presented to the museum by Queen Christina of Sweden.
- Two are conserved at the Museum of Natural History of Clermont-Ferrand, Blaise Pascal's birthplace.
- One, presented by the queen of Poland, is held at the Museum of Dresden.
- One forms part of the company IBM's collections. IBM manufactured a hundred reproductions in the 1960s as gifts to customers, and these replicas are occasionally found in antique shops.
- One of the machines is held in a private collection.

Although the basic operating principle is the same for all the machines, the exact craftsmanship varies from one to another, indicating that there were several models: decimal machines for calculation, surveyance machines, accountancy machines (a 10-digit wheel for pounds, a 20-digit wheel for sols, and 12-digit wheels for deniers).

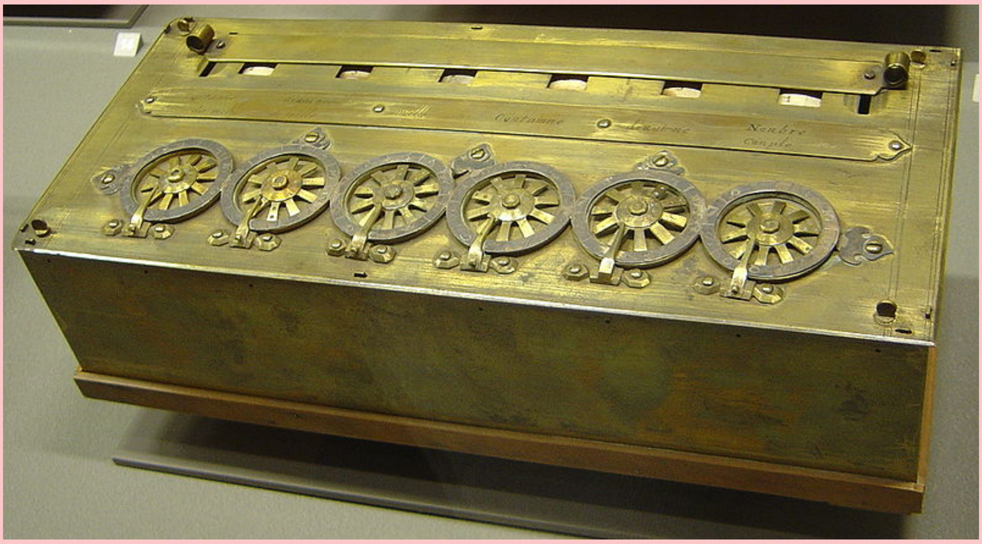

Figure 1*: One of the original Pascalines conserved at the Musée des Arts et Métiers** *(photo WikiCommons author David Monniaux).**

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To support his claim that the machine never makes a mistake, Pascal evokes the approval of "those who, in Paris, are the most versed in mathematics". Why must one be well versed in mathematics to verify the calculations? Pascal does not say. But, no doubt for his contemporaries, addition and other basic arithmetical operations were an essential part of mathematics. A sentiment, indeed, that remains much the same a few centuries later.

THE REMAINDER: A PROBLEM SOLVED WITH APLOMB

To fine-tune his machine, the principal difficulty Pascal had to resolve was the remainder. If there is no remainder, a simply designed drive mechanism can add numbers and display the result: the addition of units, tens and hundreds are independent. Such additions can be performed by turning the appropriate wheel (one for units, one for tens, etc.), which itself is independent from the others. But if, during an operation, the unit wheel reaches 9 and has to be increased by an increment of one unit, a mechanism is needed to advance the tens wheel by one unit. What's more, this operation must be performed in series when, for example, the operation process reaches 999, and one unit must be added to produce 1000…

Pascal's genius is revealed in the solution he devised to resolve this problem: a system of carry levers (*sautoirs*) arranged in series. This solution has survived the test of time. An analogous system was used in numeric calculators until the advent of digital calculators; a similar, though not exactly analogous, principle is used is many counting mechanisms, notably mileage counters in cars.

From counters to microelectronic components
via the Pascaline

Pascal addresses (D2) his reader: the machine "rids you of work that before left you mentally tired, when you used the counter or the quill". "Quill" refers to the posture adopted when recording operations by hand; "counter" refers to the use of game tokens, such as those found in Chinese abacuses, by merchants and financiers to perform their calculations.
While it was in the Pascaline that the calculation process was mechanised for the first time, like the quill and counter the machine used the principles of inscription and the remainder to perform additions.
The same mental process is still used for calculations in our modern-day calculators and computers: thus, to add two octet binary numbers (eight bits 0 or 1), a "full-adder 8 bits" component, using its

transistors' logical truth tables, will also add the successive bits from right to left by recording the result of each basic addition ("inscription") and making the remainder from 0 or 1.

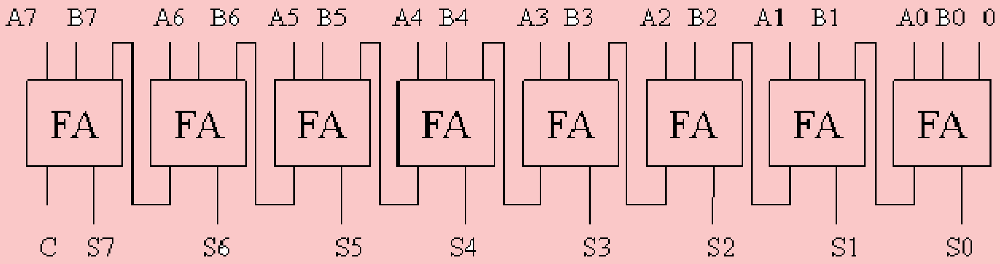

Figure 2 : ***Logical schema of a "full-adder"*** *(an electronic component for addition). To add the two octets* A0A1A2A3A4A5A6A7 *and* B0B1B2B3B4B5B6B7*, the component – like the quill and Pascal's machine – performs basic additions from right to left, adding a* S_i *each time and reinjecting – in series – the "remainder" into the following basic addition.*

For subtraction, Pascal used a simple method – complementary addition – combined with a clever double-display system.

It was therefore possible to use the machine to perform multiplications in the form of a sequence of additions, and divisions in the form of a sequence of multiplications and subtractions. But the calculation became very long very quickly, especially for divisions.

BEWARE OF COUNTERFEITS!

In his *Avis*, Pascal emphasises the fact that the development then manufacture of a machine like his requires a theoretical approach:

> *It is absolutely impossible for humble artisans, however skilled they are in their art, to perfect a new piece of machinery that – like this one – involves complicated procedures, without the aid of a person who, by the rules of theory, gives them the measurements and proportions of all the parts that it must contain.*

Pascal underlines this point to put his reader on guard against "poor copies of this machine that may be produced by presumptuous artisans":

> *I have reason to give you this last warning, after seeing with my own eyes a faulty execution of my thought by a craftsman of the town of Rouen, a clockmaker by profession* […] *but as the fellow had no other talent than that of skilfully handling his tools, and did not even know if the geometry*

and mechanics were pure […] *he produced a machine* […] *so internally imperfect that it was of no use whatsoever.*

As for counterfeits:

They display a little monster lacking its main limbs, while the others are misshapen and utterly out of proportion […] *The appearance of this little runt was most displeasing to me.*

Aside from these rather harsh words, Pascal has a very precise idea of the relationship between the designer and the manufacturer, the architect and the mason:

[…] *for new inventions, it is absolutely necessary that art be assisted by theory until usage has made the rules of theory so banal that they are finally reduced to art and continual repetition has bestowed on artisans the habit of following and practising these rules*[1] *confidently.*

But, here again, Pascal does not expand on the reasons why, in his opinion, the assistance of a person who knows the "rules of theory" is necessary. This is no doubt what his own experience had taught him, and it is a shame he does not say more, because the connection is not immediately obvious. Theoretically speaking, the Pascaline's design and mode of functioning are very simple: perhaps Pascal is alluding to the reasoning and calculations he had to carry out to determine the size of the parts and their arrangement.

A COMPLEX MACHINE FOR A SIMPLE USE

Though his text was intended to extol the virtues of his machine, Pascal makes no attempt to hide what could be considered as a defect: the fact that the machine contains many different parts. He pre-emptively silences his detractors:

I know there a many people who make it their business to find fault with everything, and among those some may be found who tell you that this machine could be less composed[2]*.*

Pascal also insists on the fact that this complicated machine required a great deal of effort on his part. But he stresses that this complexity is necessary for the machine to be endowed with all the qualities he has enumerated, in particular its simplicity and robustness. And he adds:

1. What we call the "rules of the art" when referring to the work of an artisan, notably. As the etymology of the word shows and as Pascal reminds us, the artisan, like the artist, practises an art.
2. In modern parlance "less composed" can be understood as "simpler". The fact that the machine is "composed" is related to an important point of ergonomy: the numbers are set on the upper surface of the machine and not at the front, which means that users have both the movement and the result in view, but at the price of a return mechanism at 90° to the movement, which makes the machine more complex (see Figure 1 in the Annex).

> *… in which you will notice a paradox of sorts, that to make it simpler to operate, it was necessary to make the machine's design more composed.*

That is, the machine must be complicated if it is to be simple to use. Exactly the same goes for modern software, whose increasing complexity is designed to make computers more user-friendly.

THE *PRIVILÈGE* SYSTEM: THE FORERUNNER TO THE PATENT

Paradoxically, Pascal provides no detail in his *Avis* on the internal mechanisms of his machine. He explains that it would be too complicated to do so in writing, while an oral presentation would be very straightforward. The argument, it has to be said, hardly seems convincing. Perhaps Pascal simply wished to avoid revealing his secrets? It was not until a century later, thanks to plates published in the *Encyclopédie*, that the public would discover the internal mechanisms of the Pascaline.

Pascal also wanted to protect his machine from counterfeits. In 1649 he obtained a *privilège* signed by the king, forbidding – and punishing by a heavy fine – the manufacture of an arithmetical machine. This is the same kind of protection provided nowadays by patents, which did not exist at the time.

Pascal pays tribute to Chancellor Séguier for having obtained this *privilège* for him, and continues his tirade against his counterfeiters:

> *(…) [the chancellor] saw fit to pluck out evil at its root and prevent a course of events that could have occurred to the detriment of my reputation and to the disadvantage of the public, by graciously granting me an exceptional privilege, which stifles before birth these illegitimate runts that could be engendered elsewhere than in the legitimate and necessary alliance of theory and art.*

Pascal gives two reasons justifying this *privilège*. The first is to protect his machine from substandard copies that would prove injurious to him. Those who came into contact with these copies and saw their defects might believe that Pascal's original machine suffered from the same shortcomings. As was mentioned above, Pascal himself had come across one of these deficient copies.

The second reason is the high cost of the machine. The *privilège* is intended to give Pascal time to reduce this cost while preserving the qualities of his machine. The machine contained many parts, which had to be manufactured with great precision. At the time, everything had to be done by hand, and it is not surprising that the machine was very costly. Indeed, ultimately it was the cost of the machine that prevented it becoming widely used. Pascal had forgotten, or underestimated, this aspect of things. Furthermore, it should be noted that Pascal never mentions the matter of cost in his *Avis*.

PASCAL'S MACHINE: THE FIRST MECHANISED CALCULATION PROCESS

What remains most interesting and striking in Pascal's text, however, is his conceptualisation of his invention, the first in a series of machines designed to compensate for the weakness of human memory, or as Pascal says, "relieve [man] from the failure of memory":

> *Likewise you know that when calculating by hand, it is necessary at every moment to retain or borrow the necessary numbers, and how many errors slip in during these operations if done by anyone other than the very experienced, and how this wearies the mind in no time at all. This machine delivers he who uses it from such vexation; it suffices that he have judgement, for the machine relieves him of the failure of memory.*

These words are visionary and premonitory in tone: while ultimately, only a limited number of machines would be manufactured – despite the efforts of their inventor – the machine guaranteed Pascal certain celebrity at a very young age, and even today remains a testimony to the scope of his genius.

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(March 2009)

(translated in English by Helen Tomlinson, published June 2014)

Annex: A brief explanation of how the Pascaline works

Detailed descriptions of how the Pascaline works can be found on the Internet (some websites are indicated in the bibliography). Here we will provide a brief overview of how to set digits, the mechanism, and addition.

STAGE 1: INSCRIBING A NUMBER

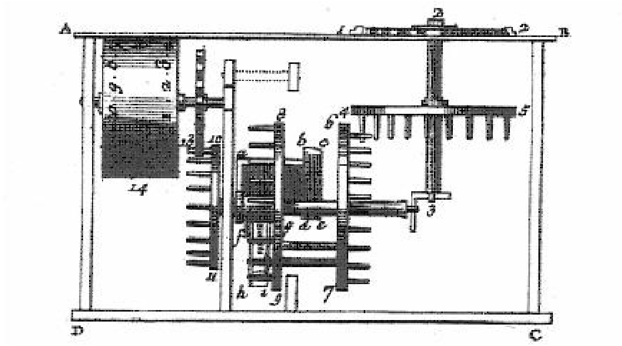
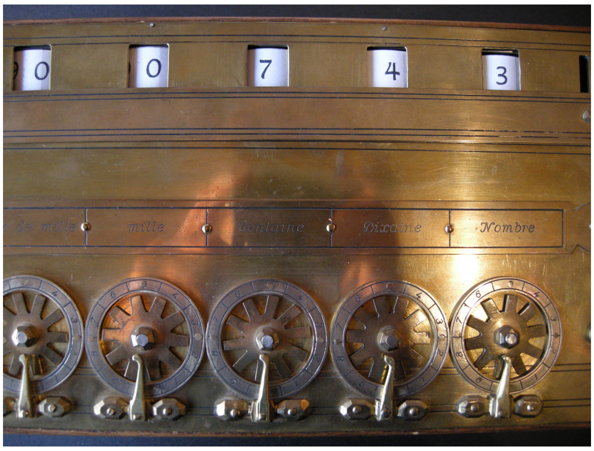
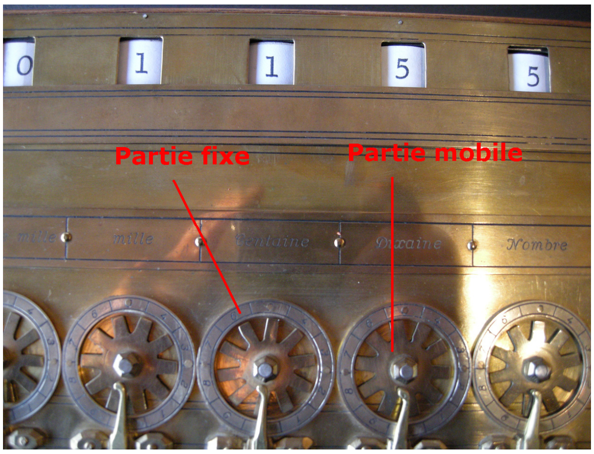


Figure 1: *The machine is shown in the top left. The six cog-wheels are called inscriptors (below, a photograph by André Devaux). By turning a cog-wheel (with a buffer stop below each wheel, a little bit like in old telephone dials, though here the wheels turn clockwise), a number is inscribed on the dial above the cog-wheel (cf. the six grooves on the display in the top-left photo). In the top-right illustration, the gear mechanism between the inscriptor and the display drum: in this figure, the AB plane shows the surface of the machine: the inscriptor (the wheel to the left of point B) on the right-hand side transmits the movement to the drum (to the right of point A).*

STAGE 2: ADDITION



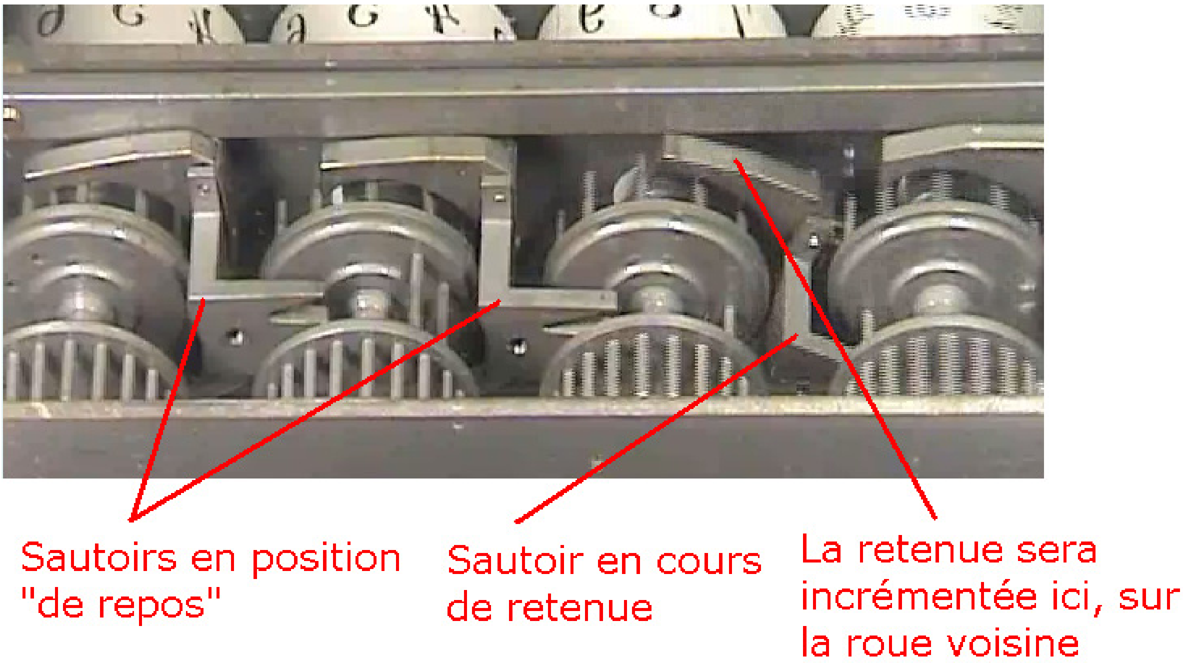


<u>Figure 2:</u> Addition 743 + 412

On the right: as in old telephone dials, the right-hand wheel is turned from position 3 to the buffer stop, the next wheel from position 4 to the buffer stop, and the third wheel from position 7 to the buffer stop. These are clockwise movements (unlike telephone dials): the display shows 743.

The ring around the inscriptor, where the digits are written, remains in a fixed position: only the cog-wheel turns. To add 412, one simply repeats the operation by turning the right-hand wheel from position 2 to the buffer stop, and so on. The display drum retains information in addition to what it displays, and shows the result 1155 (see next stage for the remainder in the hundreds wheel).

STAGE 3: THE REMAINDER AND THE *SAUTOIR*



Sautoirs in "rest" position

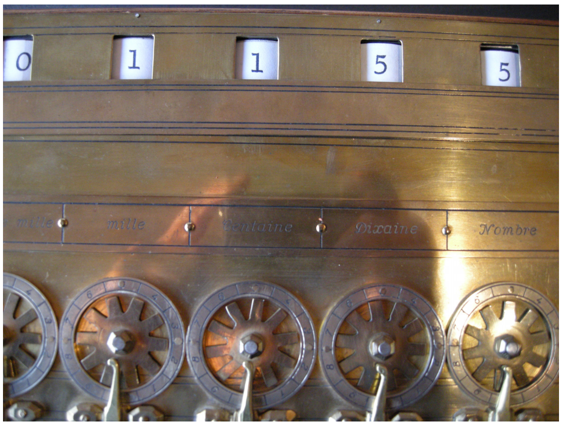
Sautoir applying the remainder

The remainder will be incremented here, on the neighbouring wheel

Figure 3: *(Image taken from a video on André Devaux's website, legends BibNum.) Let's look at what happens to the wheel on the right-hand side of the above figure (the hundreds wheel in the previous example). To apply a remainder, the wheel has activated its* sautoir *in a downward direction: the two other* sautoirs *are horizontal, in rest position. When it returns to its initial position, the* sautoir *increases the neighbouring wheel by an increment of one. In the example of addition above, the right-hand wheel is for the hundreds and is in motion (hence why the wheel and its display are blurred). As this addition requires a remainder, the* sautoir *increases by an increment of one the neighbouring thousands wheel (the second from the right). If we look closely (third wheel from the right), we can see two bars in the centre of the drum: when the wheel turns, the first of these bars moves into the* sautoir *pin and activates it downwards.*

This sautoir *mechanism operates in series; some operations require a "remainder transfer" (e.g. 193 + 8 = 201: the remainder in the tens is transferred to the hundreds). In this respect, the balance weights making up the* sautoirs *must increase in weight towards the units wheel for this gravity-operated remainder mechanism to work and carry out the transfer.*

STAGE 4: RESETTING THE MACHINE TO ZERO



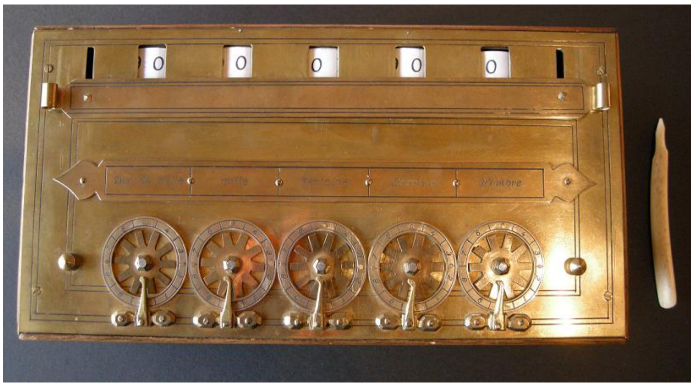


***Figure 4:* Resetting the machine to zero.**

After the addition (the result 1155 on the left), or indeed any operation, the machine has to be reset to zero by hand, which is one of the drawbacks of the Pascaline. To do this, the mobile part of each wheel is turned clockwise until zero is displayed. It is important to start with the wheel on the far right (else the rotation risks applying a remainder, and the resetting operation has to begin over again). Later machines, such as the addometer (USA, 20th c.), based on a system of wheels and remainders logically analogous to the Pascaline, were equipped with an automatic reset device (a pulltab). See photo below, from the collection of Yves Serra.

A.M.